

Curves with a group action and Galois covers via infinite Grassmannians

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Queremos dedicar este trabajo a nuestro amigo Sevín y su familia. De sus muchas cualidades humanas, queremos resaltar su interés por los jóvenes matemáticos y, en especial, la cálida hospitalidad con la que siempre nos acogieron en nuestras visitas a México.

ABSTRACT. The moduli functor of curves with an action of a finite group with a distinguished orbit and a trivialization along it is shown to be representable by a closed subscheme of the Sato Grassmannian. A similar result for the moduli functor of Galois covers is also given. Finally, the equations of bilinear type characterizing these subschemes are explicitly computed.

1. Introduction

Subschemes of the moduli space of curves consisting of those curves with an action of a given group have been largely studied since their knowledge sheds light on the geometry of this space. Properties of Hurwitz spaces of Galois covers have also consequences in enumerative geometry and on the class field theory. In this paper we study both objects from the point of view of the theory of infinite dimensional Grassmannians.

When dealing with the moduli space of curves admitting an action of a finite group G , the first task consists of *rigidifying* the geometric data. This will be achieved by giving an orbit, a formal trivialization along it and a group monomorphism of G into the group of $r \times r$ -matrices of quasipermutations with coefficients in the group of m -th roots of unity, $\mathbb{Q}\mathbb{P}_r^m$, where r is the number of distinct points in the orbit and $m := \frac{1}{r}|G|$ (see the subsection 2.1 for details). Denote by $\mathcal{M}_G^\infty(r)$ the corresponding moduli functor.

The representability of $\mathcal{M}_G^\infty(r)$ by a closed subscheme of the Sato Grassmannian is shown in Theorem 2.2 and its proof lies on the properties of the Krichever map.

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Further, its points admit a Mulase-like characterization in terms of finite dimensional orbits (see Theorem 2.4). These results are a generalization of [GMP] where the case of groups of prime order was carried out.

The third section is devoted to the moduli space of Galois covers. Its points are the points of the Hurwitz space of [MP2] such that the cover is Galois. Firstly, we characterize its points as those points of the Hurwitz space such that its stabilizer under the action of QP_r^m has order equal to $r \cdot m$ (Theorem 3.4). Its representability as a subscheme of the Sato Grassmannian is a consequence of this characterization (Theorem 3.5). Finally, we build the moduli space of Galois covers with fixed group and the moduli space of Galois cover of a fixed curve.

The last section is concerned with the explicit computation of equations for the two moduli spaces previously studied. In order to do that, an explicit expression of the action of QP_r^m on the Baker-Akhiezer function is needed. Then, the equations of bilinear type can be easily written down (Theorems 4.2 and 4.3). Note that standard techniques allows one to translate these equations into a hierarchy of p.d.e. for the τ -function.

The following notation will be used; for a scheme X , the corresponding functor of points will be denoted by $\underline{X}$.

2. Moduli of curves with a group action

This section is concerned with the study of the moduli space of curves carrying an action of the group G . We begin studying the geometric case.

2.1. Geometric setup. Let Y be an integral complete smooth curve over an algebraically closed field k of characteristic 0. Let G be a finite group of order n and $G \hookrightarrow \mathrm{Aut}(Y)$ be an action of G on Y . Let $\bar{y}$ be an orbit of G consisting of r pairwise distinct smooth points $\{y_1, \dots, y_r\}$.

Let H denote the stabilizer of the point y_1 . Since it acts on the tangent space of Y at y_1 , one gets

$$H \hookrightarrow \mathrm{Aut}(T_{y_1}Y) \simeq k^* .$$

It follows that H is cyclic and that this map yields a generating character ξ of H . Furthermore, since G acts transitively on the orbit $\bar{y}$, the stabilizers of the points y_j are conjugate to H and $|H| = \frac{n}{r} =: m$.

Now, choose elements $g_2, \dots, g_r \in G$ such that $g_i(y_1) = y_i$ and fix a formal trivialization at y_1

$$t_1 : \widehat{\mathcal{O}}_{Y, y_1} \xrightarrow{\sim} k[[z]] .$$

Then, these data give rise to a formal trivialization at y_i given by $t_i := t_1 \circ g_i^*$ where $g_i^* : \widehat{\mathcal{O}}_{Y, y_i} \xrightarrow{\sim} \widehat{\mathcal{O}}_{Y, y_1}$. Putting this trivializations together, one obtains an isomorphism

$$t_{\bar{y}} : \widehat{\mathcal{O}}_{Y, \bar{y}} \xrightarrow{\sim} k[[z_1]] \times \dots \times k[[z_r]] =: V^+ .$$

Let us make explicit the induced action of G on V^+ via $t_{\bar{y}}$. Given an element $g \in G$ and $1 \leq i \leq r$, there exists j such that $g(y_i) = y_j$. Hence $g_j^{-1}gg_i \in H$ and

$$t_1 \circ (g_j^{-1}gg_i)^* = \xi(g_j^{-1}gg_i)t_1$$

where $\xi(g_j^{-1}gg_i)$ is a m -root of unity. Translating this fact in terms of the z 's, one gets $g(z_i) = \xi(g_j^{-1}gg_i)z_j$, which describes the action of g on V^+ .

Let us introduce the group of *quasi-permutation matrices* $r \times r$ with coefficients in the group of m -root of unity by

$$\mathrm{QP}_r^m := \left\{ \begin{array}{l} \text{invertible } r \times r\text{-matrices such that there is} \\ \text{exactly one non-zero entry on each column and} \\ \text{on each row and all entries are } m\text{-roots of unity} \end{array} \right\}.$$

Observe that an element $(a_{ij}) \in \mathrm{QP}_r^m$ defines an automorphism of k -algebras of V^+ (and of V) as follows

$$(2.1) \quad z_j \longmapsto \sum_i a_{ij} z_i.$$

Then, the above arguments show that the induced action of G on V^+ factorizes by a morphism

$$\rho: G \hookrightarrow \mathrm{QP}_r^m.$$

Summing up, given $(Y, \bar{y}, G \hookrightarrow \mathrm{Aut}(Y))$ as above, there exists a formal trivialization $t_{\bar{y}}$ along $\bar{y}$, which induces a group monomorphism $\rho: G \hookrightarrow \mathrm{QP}_r^m$ such that the action on $\mathrm{Spf} V^+$ is transitive.

2.2. Formal Geometry and Infinite Grassmannians. Having in mind the above explained facts, we will introduce the necessary tools for the study of the moduli space of curves with an action of a given group G in terms of infinite Grassmannians. To this end, results of [MP, MP2, GMP] will be recalled and adapted to the present situation.

Consider the $k((z))$ -algebra

$$V := k((z_1)) \times \cdots \times k((z_r))$$

where z goes to $(z_1^m, \dots, z_r^m)$. Consider the subspaces $V^+ := k[[z_1]] \times \cdots \times k[[z_r]]$ and $V^- := z_1^{-1}k[[z_1^{-1}]] \times \cdots \times z_r^{-1}k[[z_r^{-1}]]$.

Recall that Γ is the connected component of 1 of the formal group scheme representing the functor

$$\begin{array}{ccc} \text{category of} & & \text{category} \\ \text{formal } k\text{-schemes} & \rightsquigarrow & \text{of groups} \end{array}$$

$$S \rightsquigarrow k((z)) \hat{\otimes}_k H^0(S, \mathcal{O}_S)^* = \left\{ \begin{array}{l} \text{invertible elements} \\ \text{of } k((z)) \hat{\otimes}_k H^0(S, \mathcal{O}_S) \end{array} \right\}.$$

One has a decomposition $\Gamma = \mathcal{J}(\widehat{C}) \times \bar{\Gamma}^+$ where $\mathcal{J}(\widehat{C})$, which is called the jacobian of $\widehat{C}$, represents the subfunctor of Γ whose points are those elements lying in $k[[z^{-1}]] \hat{\otimes}_k H^0(S, \mathcal{O}_S)$ whose constant term is equal to 1; and $\bar{\Gamma}^+$ represents the subfunctor whose points are those elements lying in $k[[z]] \hat{\otimes}_k H^0(S, \mathcal{O}_S)$ whose constant term is invertible.

For the case of the formal cover, Γ_V is defined as the connected component of 1 of the formal group scheme representing the functor

$$S \rightsquigarrow V \hat{\otimes}_k H^0(S, \mathcal{O}_S)^* = \left\{ \begin{array}{l} \text{invertible elements} \\ \text{of } V \hat{\otimes}_k H^0(S, \mathcal{O}_S) \end{array} \right\}.$$

Similarly to the case of Γ , this formal group is decomposed as $\Gamma_V = \mathcal{J}(\widehat{C}_V) \times \bar{\Gamma}_{V^+}$, according to the sign of the powers. It is worth pointing out that, in particular, the set of R -valued points of $\mathcal{J}(\widehat{C}_V)$ is given by

$$\left\{ \left\{ \exp \left(\sum_{j>0} t_j^{(i)} z_i^{-j} \right) \right\} \in \prod_{1 \leq i \leq r} R((z_i)) \text{ such that } t_j^{(i)} \in \mathrm{Rad}(R) \text{ for all } i, j \right\}$$

and those of $\bar{\Gamma}_{V^+}$ by

$$\left\{ \left\{ \exp\left(\sum_{j \geq 0} t_j^{(i)} z_i^j\right) \right\} \in \prod_{1 \leq i \leq r} R((z_i)) \text{ such that } t_j^{(i)} \in R \text{ for all } i, j \right\}.$$

The infinite Grassmannian of (V, V^+) is a k -scheme whose set of rational points is

$$\left\{ \begin{array}{l} \text{subspaces } U \subset V \text{ such that } U \rightarrow V/V^+ \\ \text{has finite dimensional kernel and cokernel} \end{array} \right\}.$$

The connected components of this scheme are denoted by $\text{Gr}^m(V)$ ($m \in \mathbb{Z}$) where m is the Poincaré-Euler characteristic of $U \rightarrow V/V^+$.

There is a canonical line bundle on this scheme, which is given by the determinant of the complex

$$(2.2) \quad \mathcal{L} \rightarrow V/V^+$$

where $\mathcal{L}$ is the universal submodule.

Recall that $\text{Gr}(V)$ is endowed with an action of the group Γ_V :

$$(2.3) \quad \Gamma_V \times \text{Gr}(V) \longrightarrow \text{Gr}(V)$$

since the elements of Γ_V act by homotheties on V .

We will fix a group monomorphism

$$(2.4) \quad \rho: G \hookrightarrow \text{QP}_r^m$$

(with $n := |G| = mr$) such that the induced action of G on $\text{Spf } V^+$ is transitive.

On the other hand, observe that from the action of G on V (via ρ) one deduces an action of G on $\text{Gr}(V)$

$$(2.5) \quad \begin{array}{l} G \times \text{Gr}(V) \longrightarrow \text{Gr}(V) \\ (g, U) \longmapsto \rho(g)(U) \end{array}$$

that preserves the determinant bundle. Since $\text{Gr}(V)$ is separated, it follows that the subfunctor of fixed points of $\text{Gr}(V)$ under the action of G

$$\text{Gr}(V)^{\rho(G)} := \{U \in \text{Gr}(V) \text{ such that } \rho(g)(U) = U \text{ for all } g \in G\}$$

is representable by a closed subscheme of $\text{Gr}(V)$.

2.3. Moduli space. In this part, we continue with a given group G and a given $\rho: G \hookrightarrow \text{QP}_r^m$ as in (2.4). We focus on the study of the moduli space of curves with an action of G such that the induced action on the completion is given through the morphism ρ .

DEFINITION 2.1. The moduli functor of curves with an action of G of the type ρ and an orbit consisting of r points, $\mathcal{M}_{G, \rho}^\infty(r)$, is the contravariant functor on the category of k -schemes whose set of S -valued points is given by the set of equivalence classes of data

$$(Y, \bar{y}, t_{\bar{y}}, \rho_Y) := Y, \{y_1, \dots, y_r\}, \{t_1, \dots, t_r\}, \rho_Y: G \hookrightarrow \text{Aut}_S(Y)$$

where

- (1) $p_Y: Y \rightarrow S$ is a proper and flat morphism whose fibres are geometrically reduced curves,
- (2) ρ_Y is a group monomorphism from G into $\text{Aut}_S(Y)$,
- (3) $\{y_1, \dots, y_r\}$ are disjoint smooth sections of p_Y which are Cartier divisors of Y and they are an orbit under the action of G ,

- (4) for each closed point $s \in S$ it holds that $y_i(s)$ is a smooth point of the fibre $p_Y^{-1}(s) = Y_s$ for all i , and that each irreducible component of Y_s contains a point $y_i(s)$ for some i ,
- (5) there exists formal parameters t_i along the sections y_i such that they induce an isomorphism:

$$t_{\bar{y}}: \widehat{\mathcal{O}}_{Y, \bar{y}(S)} \xrightarrow{\sim} V^+ \widehat{\otimes} \mathcal{O}_S$$

which is compatible with respect to the actions of G on $\widehat{\mathcal{O}}_{Y, \bar{y}(S)}$ and on $V^+ \widehat{\otimes} \mathcal{O}_S$ (via ρ),

- (6) $(Y, \bar{y}, t_{\bar{y}}, \rho_Y)$ and $(Y', \bar{y}', t_{\bar{y}'}, \rho_{Y'})$ are said to be equivalent when there is an G -equivariant morphism of S -schemes

$$Y \xrightarrow{\sim} Y'$$

compatible with all the data.

Observe that $\underline{\mathcal{M}}_{G, \rho}^\infty(r)$ is a subfunctor of $\underline{\mathcal{M}}^\infty(r)$, where the closed points of $\underline{\mathcal{M}}^\infty(r)$ ([MP2]) correspond to the classes of sets of data

$$(Y, \bar{y}, t_{\bar{y}}) := (Y, \{y_1, \dots, y_r\}, \{t_1, \dots, t_r\})$$

of geometrically reduced curves with r marked pairwise distinct smooth points $\{y_1, \dots, y_r\}$ and formal parameters $\{t_1, \dots, t_r\}$ at these points and such that any irreducible component contains at least one of the marked points.

We define the Krichever map for $\underline{\mathcal{M}}^\infty(r)$ as the morphism of functors

$$\text{Kr}: \underline{\mathcal{M}}^\infty(r) \longrightarrow \underline{\text{Gr}}(V)$$

that assigns to $(Y, \bar{y}, t_{\bar{y}}, \rho_Y)$ the submodule

$$t_{\bar{y}} \left(\varinjlim_{n \geq 0} (p_Y)_* \mathcal{O}_Y(n \cdot \bar{y}) \right) \subset V \widehat{\otimes} \mathcal{O}_S.$$

Recall that, thanks to the Krichever map, one shows that $\mathcal{M}^\infty(r)$ might be identified with a closed subscheme of $\text{Gr}(V)$. The points of $\text{Gr}(V)$ lying on $\mathcal{M}^\infty(r)$ are characterized as those subspaces $U \subseteq V$ such that $U \cdot U = U$ ([MP2], Theorem 4.3).

THEOREM 2.2. *The functor $\underline{\mathcal{M}}_{G, \rho}^\infty(r)$ is representable by a closed subscheme $\mathcal{M}_{G, \rho}^\infty(r)$ of $\text{Gr}(V)$.*

PROOF. It holds that $\underline{\mathcal{M}}_{G, \rho}^\infty(r) = \underline{\mathcal{M}}^\infty(r) \times_{\underline{\text{Gr}}(V)} \underline{\text{Gr}}(V)^{\rho(G)}$.

REMARK 2.3. If one would like to become independent of ρ , i.e. to consider curves with an action of G , then one could study the scheme $\mathcal{M}_G^\infty(r) := \cup_\rho \mathcal{M}_{G, \rho}^\infty(r)$ where ρ runs over the set of group monomorphisms $G \hookrightarrow \text{QP}_r^m$ such that the action induced by G on $\text{Spec } V^+$ is transitive. Note that the set of these ρ is finite.

In order to give a Mulase-like characterization of those points of the Sato Grassmannian that arise from geometric data over an algebraic curve with automorphisms, we need an extension of the Krichever construction.

Consider data

$$(Y, \rho_Y, \bar{y}, t_{\bar{y}}, L, \phi_{\bar{y}})$$

where $(Y, \rho_Y, \bar{y}, t_{\bar{y}})$ belongs to $\underline{\mathcal{M}}_{G, \rho}^\infty(r)$, L is a rank 1 torsion free sheaf on Y such that $\phi_{\bar{y}}$ is a formal trivialization of L along $\bar{y}(S)$, that is, an isomorphism $\phi_{\bar{y}}: \widehat{L}_{\bar{y}(S)} \simeq \widehat{\mathcal{O}}_{Y, \bar{y}(S)}$.

Then, given such data $(Y, \rho_Y, \bar{y}, t_{\bar{y}}, L, \phi_{\bar{y}})$ we define the Krichever map by

$$\mathrm{Kr}(Y, \rho_Y, \bar{y}, t_{\bar{y}}, L, \phi_{\bar{y}}) := (t_{\bar{y}} \circ \phi_{\bar{y}}) \quad H^0(Y - \bar{y}, L) \subset V \hat{\otimes} \mathcal{O}_S.$$

Note that $\mathcal{M}_{G, \rho}^\infty(r)$ is contained in the image of the Krichever map since the points of the form $(Y, \rho_Y, \bar{y}, t_{\bar{y}}, \mathcal{O}_Y, \mathrm{Id})$ are precisely the points of $\mathcal{M}_{G, \rho}^\infty(r)$.

Let us consider the following embedding of $\mathrm{Gr}(V)$ into $\mathrm{Gr}(V)^n := \mathrm{Gr}(V) \times \dots \times \mathrm{Gr}(V)$ (where $|G| = n$)

$$\begin{aligned} \mathrm{Gr}(V) &\hookrightarrow \mathrm{Gr}(V)^n \\ U &\mapsto U_{\rho(G)} := (\rho(g_1)(U), \dots, \rho(g_n)(U)) \end{aligned}$$

where $\{g_1, \dots, g_n\}$ denote the elements of G .

Observe that Γ_V acts on $\mathrm{Gr}(V)^n$

$$\begin{aligned} \mu^n: \Gamma_V \times \mathrm{Gr}(V)^n &\longrightarrow \mathrm{Gr}(V)^n \\ (f, (U_1, \dots, U_n)) &\mapsto (fU_1, \dots, fU_n). \end{aligned}$$

And, for a closed point $U \in \mathrm{Gr}(V)$, denote by $\Gamma_V(U_{\rho(G)})$ the orbit of $U_{\rho(G)} \in \mathrm{Gr}(V)^n$ under the action of Γ_V .

THEOREM 2.4. *Let $U \in \mathrm{Gr}(V)$ be a closed point.*

Then, the vector space $T_{U_{\rho(G)}}(\Gamma_V(U_{\rho(G)})/\bar{\Gamma}_V^+)$ is finite dimensional if and only if there exists data $(Y, \rho_Y, \bar{y}, t_{\bar{y}}, L, \phi_{\bar{y}})$ with $(Y, \rho_Y, \bar{y}, t_{\bar{y}}) \in \mathcal{M}_{G, \rho}^\infty(r)$ such that $U = \mathrm{Kr}(Y, \rho_Y, \bar{y}, t_{\bar{y}}, L, \phi_{\bar{y}})$.

PROOF. Let us sketch the proof which is based in the proof of Theorem 4.12 of [GMP]. Observe that

$$T_{U_{\rho(G)}}(\Gamma_V(U_{\rho(G)})/\bar{\Gamma}_V^+) \simeq V/(A_U + V^+),$$

where A_U is the k -algebra defined by

$$A_U := \{v \in V \text{ such that } v \cdot \rho(g_i)(U) \subseteq \rho(g_i)(U) \quad \forall i\}.$$

By [LM], the hypothesis that $\dim_k T_{U_{\rho(G)}}(\Gamma_V(U_{\rho(G)})/\bar{\Gamma}_V^+) < \infty$ is equivalent to the fact that $A_U \in \mathrm{Gr}(V)$; that is, A_U corresponds to data $(Y, \rho_Y, \bar{y}, t_{\bar{y}}) \in \mathcal{M}_{G, \rho}^\infty(r)$. Having in mind that the points of $\mathrm{Gr}(V)$ which are A_U -modules correspond to pairs $(L, \phi_{\bar{y}})$ over $(Y, \rho_Y, \bar{y}, t_{\bar{y}})$ and that U is A_U -module, the statement follows.

This type of characterizations dates back to the approach of Mulase ([M]). For a detailed study of $\Gamma_V(U_{\rho(G)})$ and its geometrical interpretation we address the reader to [M, GMP].

3. Moduli of Galois covers

Let us recall from [MP] some facts on Hurwitz spaces; i.e. moduli spaces of covers. Let $\bar{\mathcal{H}}^\infty(m, \overset{r}{\cdot}, m)$ be the functor parametrizing covers $\pi: Y \rightarrow X$ of degree rm together with a point and a formal parameter downstairs (x, t) and r pairwise distinct smooth points and parameters upstairs $(\bar{y} = \{y_1, \dots, y_r\}, t_{\bar{y}} = (t_1, \dots, t_r))$ such that $\pi^{-1}(x) = m\bar{y}$ and $t_i^m = (\pi^*t)_{y_i}$. We require this data to satisfy one technical condition; namely, $\mathrm{Tr}(H^0(Y - \bar{y}, \mathcal{O}_Y)) \subseteq H^0(X - x, \mathcal{O}_X)$ where Tr is the trace map of the total quotient ring of Y as an algebra over the total quotient ring of X .

Furthermore, this functor is characterized as follows

THEOREM 3.1 ([MP2]). *The forgetful map $\bar{\mathcal{H}}^\infty(m, \overset{r}{\cdot}, m) \rightarrow \mathcal{M}^\infty(r)$ is an injection.*

Further, a closed point $U \in \mathcal{M}^\infty(r)$ belongs to $\bar{\mathcal{H}}^\infty(m, \overset{r}{\cdot}, m)$ if and only if $\text{Tr}(U) \subseteq U$ (Tr denotes the trace map $V \rightarrow k((z))$ as finite $k((z))$ -algebra).

It follows that it is representable by a closed subscheme of $\mathcal{M}^\infty(r)$ which will be denoted by $\bar{\mathcal{H}}^\infty(m, \overset{r}{\cdot}, m)$.

For the sake of clarity, the study is focused now on smooth curves. Let $\mathcal{Y}$ and $\mathcal{X}$ be the universal curves over $\bar{\mathcal{H}}^\infty(m, \overset{r}{\cdot}, m)$; that is, the fiber of $\mathcal{Y}$ (resp. $\mathcal{X}$) over a point $s = (Y, X, \pi, x, t_x, \bar{y}, t_{\bar{y}})$ is Y (resp. X). Then, we denote by $\mathcal{H}^\infty(m, \overset{r}{\cdot}, m)$ the open subscheme of $\bar{\mathcal{H}}^\infty(m, \overset{r}{\cdot}, m)$ consisting of those points s such that $\mathcal{Y}_s$ and $\mathcal{X}_s$ are smooth connected curves.

DEFINITION 3.2. The functor $\underline{\mathcal{H}}_{\text{Gal}}^\infty(m, \overset{r}{\cdot}, m)$ is the subfunctor of $\bar{\mathcal{H}}^\infty(m, \overset{r}{\cdot}, m)$ consisting of those $(Y, X, \pi, x, t_x, \bar{y}, t_{\bar{y}})$ such that $\pi: Y \rightarrow X$ is a Galois cover.

Since the points of $\mathcal{M}^\infty(r)$ corresponds to those subspaces of V that are subalgebras and the group QP_r^m acts on V by $k((z))$ -algebra automorphisms, we obtain an action of QP_r^m on $\mathcal{M}^\infty(r)$. For a point $U \in \mathcal{M}^\infty(r)$ the stabilizer under this action will be denoted by $\text{Stab}(U)$.

PROPOSITION 3.3. Let $(Y, X, \pi, x, t_x, \bar{y}, t_{\bar{y}}) \in \mathcal{H}^\infty(m, \overset{r}{\cdot}, m)$ and let $U \in \mathcal{M}^\infty(r)$ be its image under the Krichever map, that is, $U = \text{Kr}(Y, \bar{y}, t_{\bar{y}})$. Then

$$\text{Stab}(U) \subseteq \text{Aut}(Y/X).$$

PROOF. Let $U \in \mathcal{H}^\infty(m, \overset{r}{\cdot}, m)$ be a closed point. Then $\text{Tr}(U) = U \cap k((z))$ is the ring of the affine curve $X - x$. Since QP_r^m acts on V by $k((z))$ -algebra automorphisms, the stabilizer of U acts trivially on $U \cap k((z))$ and, therefore, $\text{Stab}(U) \subseteq \text{Aut}(Y/X)$.

The points of the functor $\underline{\mathcal{H}}_{\text{Gal}}^\infty(m, \overset{r}{\cdot}, m)$ are characterized as follows

THEOREM 3.4. *A point U of $\underline{\mathcal{H}}_{\text{Gal}}^\infty(m, \overset{r}{\cdot}, m)$ lies on $\underline{\mathcal{H}}_{\text{Gal}}^\infty(m, \overset{r}{\cdot}, m)$ if and only if $|\text{Stab}(U)| = r \cdot m$.*

PROOF. Let $(Y, X, \pi, x, t_x, \bar{y}, t_{\bar{y}})$ be a point in $\underline{\mathcal{H}}_{\text{Gal}}^\infty(m, \overset{r}{\cdot}, m)$ and let $g \in G := \text{Aut}(Y/X)$. Since $t_i^m = (\pi^* t_x)_{y_i}$ and $\pi^* t_x$ is invariant by g , it follows that g permutes t_i^m ; that is, the action on t_i yields a monomorphism $\rho: G \hookrightarrow \text{QP}_r^m$. Having in mind that $\bar{y}$ is an orbit of g , it follows that the action of G on $\text{Spf } V^+$ induced by ρ is transitive.

Furthermore, it is straightforward that

$$(3.1) \quad \rho(G) \subseteq \text{Stab}(U)$$

as subgroups of QP_r^m . Then, by Proposition 3.3 one gets $\rho(G) \subseteq \text{Stab}(U) \subseteq G$. Since $|G| = r \cdot m$, the claim follows.

Conversely. Note that $\text{Stab}(U) \subseteq \text{Aut}(Y/X)$ and that $|\text{Aut}(Y/X)| \leq \deg \pi = r \cdot m$. The hypothesis implies that $|\text{Aut}(Y/X)| = \deg \pi = r \cdot m$, so $\pi: Y \rightarrow X$ is a Galois cover.

Now, we can show the representability of the functor of Galois covers.

THEOREM 3.5. *The functor $\mathcal{H}_{\text{Gal}}^\infty(m, \overset{\tau}{\dots}, m)$ is representable by a subscheme of $\text{Gr}(V)$.*

PROOF. Consider the action of QP_r^m on $\text{Gr}(V)$. Since QP_r^m is finite and $\text{Gr}(V)$ is separated, the quotient is a scheme. Then, the quotient map

$$\Phi: \text{Gr}(V) \longrightarrow \text{Gr}(V)/\text{QP}_r^m$$

is a finite morphism of degree $m^r \cdot r!$. The ramification locus of Φ , $R \subset \text{Gr}(V)$, carries a stratification

$$R = R_0 \supseteq R_1 \supseteq \dots \supseteq R_j \supseteq \dots$$

where R_j is the subscheme of R consisting of the points whose stabilizer under the action of QP_r^m has order $\geq j$.

Then, the previous characterization claims that

$$\mathcal{H}_{\text{Gal}}^\infty(m, \overset{\tau}{\dots}, m) = \mathcal{H}^\infty(m, \overset{\tau}{\dots}, m) \cap R_{r \cdot m}$$

and the statement follows.

If we want to study the subfunctor of $\mathcal{H}_{\text{Gal}}^\infty(m, \overset{\tau}{\dots}, m)$ consisting of those Galois covers that have automorphisms group isomorphic to a given group G , then one should consider

$$\mathcal{H}_G^\infty(m, \overset{\tau}{\dots}, m) := \mathcal{H}_{\text{Gal}}^\infty(m, \overset{\tau}{\dots}, m) \times_{\mathcal{M}^\infty(r)} \mathcal{M}_G^\infty(r).$$

Now, we modify slightly our approach and aim at studying Galois covers of given degree over a fixed curve.

THEOREM 3.6. *Let (X, x, t) be a rational point of $\mathcal{M}^\infty(1)$. The moduli functor of G -covers of the curve X with a fiber of type $(m, \overset{\tau}{\dots}, m)$, $\mathcal{H}_{X,G}^\infty(r)$, defined as follows*

$$\mathcal{H}_{X,G}^\infty: S \rightsquigarrow \left\{ (\mathcal{Y}, \mathcal{X}, \pi, x, t_x, \bar{y}, t_{\bar{y}}) \in \mathcal{H}_G^\infty(m, \overset{\tau}{\dots}, m)(S) \right. \\ \left. \text{such that } \mathcal{Y}/G \simeq \mathcal{X} \simeq X \times S \right\}$$

is representable by a subscheme of $\text{Gr}(V)$.

PROOF. Consider the forgetful morphism

$$\mathcal{H}_G^\infty(m, \overset{\tau}{\dots}, m) \longrightarrow \mathcal{M}^\infty(1)$$

which sends the data corresponding to a cover $(\mathcal{Y}, \mathcal{X}, \pi, x, \bar{y}, t_x, t_{\bar{y}})$ to the curve downstairs $(\mathcal{X}, x, t_x)$. Then, the fiber of the closed point (X, x, t_x) is the desired subscheme.

If we replace $\mathcal{H}_G^\infty(m, \overset{\tau}{\dots}, m)$ by $\mathcal{H}_{\text{Gal}}^\infty(m, \overset{\tau}{\dots}, m)$ in the statement above, one may study the moduli space of Galois covers of given degree over a fixed curve X .

4. Equations

In these section we aim at characterizing whenever a cover do admit an action of a given group in terms of bilinear identities; in particular, we give equations for the moduli spaces $\mathcal{M}_{G,\rho}^\infty(r)$ and $\mathcal{H}_{\text{Gal}}^\infty(m, \overset{\tau}{\dots}, m)$. To this goal, explicit formulae for the action of QP_r^m on τ -functions and Baker-Akhiezer functions are required.

The formal base curve is defined as the formal scheme $\widehat{C} := \text{Spf } k[[z]]$ and the formal Galois cover as the formal scheme $\widehat{C}_V := \text{Spf } V^+$. Recall that an element $(a_{ij}) \in \text{QP}_r^m$ acts on V^+ as a k -algebra automorphism by (2.1) and, therefore, it

induces an action on $\widehat{C}_V = \mathrm{Spf} V^+$. Let $\sigma \in \mathcal{S}_r$ be the permutation defined by g ; that is, $a_{ij} \neq 0$ if and only if $i = \sigma(j)$ or, equivalently, $g(z_j) = a_{\sigma(j)j} z_{\sigma(j)}$. In particular, $g^{-1}(z_j) = a_{j\sigma^{-1}(j)}^{-1} z_{\sigma^{-1}(j)}$.

Recall that the set of R -valued points of $\mathcal{J}(\widehat{C}_V)$ is given by

$$(4.1) \quad \left\{ \left\{ \exp\left(\sum_{j>0} t_i^{(j)} z_j^{-i}\right) \right\} \in \prod_{1 \leq j \leq r} R((z_j)) \text{ with } t_i^{(j)} \in \mathrm{Rad}(R) \forall i, j \right\}$$

and that, therefore, $\mathcal{J}(\widehat{C}_V)$ is the formal spectrum of

$$(4.2) \quad \mathcal{O}_{\mathcal{J}(\widehat{C}_V)} := k\{\{t_1^{(1)}, t_2^{(1)}, \dots\}\} \widehat{\otimes} \dots \widehat{\otimes} k\{\{t_1^{(r)}, t_2^{(r)}, \dots\}\}.$$

Since the formal scheme $\mathcal{J}(\widehat{C}_V)$ plays the role of the jacobian of $\widehat{C}_V$, the action of $g = (a_{ij}) \in \mathrm{QP}_r^m$ on $\widehat{C}_V$ transforms the elements of $\mathcal{J}(\widehat{C}_V)$ as follows

$$\begin{aligned} g\left(\left\{ \exp\left(\sum_{i>0} t_i^{(j)} z_j^{-i}\right) \right\}_{1 \leq j \leq r}\right) &= \left\{ \exp\left(\sum_{i>0} t_i^{(j)} a_{\sigma(j)j}^{-i} z_{\sigma(j)}^{-i}\right) \right\}_{1 \leq j \leq r} = \\ &= \left\{ \exp\left(\sum_{i>0} t_i^{(\sigma^{-1}(j))} a_{j\sigma^{-1}(j)}^{-i} z_j^{-i}\right) \right\}_{1 \leq j \leq r}. \end{aligned}$$

Equivalently, the corresponding automorphism g^* on $\mathcal{O}(\mathcal{J}(\widehat{C}_V))$ is

$$(4.3) \quad \begin{aligned} \mathcal{O}(\mathcal{J}(\widehat{C}_V)) &\xrightarrow{g^*} \mathcal{O}(\mathcal{J}(\widehat{C}_V)) \\ t_i^{(j)} &\longmapsto a_{j\sigma^{-1}(j)}^{-i} t_i^{(\sigma^{-1}(j))}. \end{aligned}$$

In particular, $(g^*)^{-1}(t_i^{(j)}) = a_{\sigma(j)j}^i t_i^{(\sigma(j))}$.

Now, following [MP2] τ -functions and Baker-Akhiezer functions will be introduced. The determinant of the morphism $\mathcal{L} \rightarrow V/V^+$ (see (2.2)) gives rise to a canonical global section

$$\Omega_+ \in H^0(\mathrm{Gr}^0(V), \mathrm{Det}_V^*).$$

The τ -function of $U \in \mathrm{Gr}^0(V)$, $\tau_U(t)$, is a function on $\mathcal{J}(\widehat{C}_V)$ and is introduced as a suitable trivialization of the restriction of the section Ω_+ to $\mathcal{J}(\widehat{C}_V)$. Let t be the set of variables $(t^{(1)}, \dots, t^{(r)})$ (where $t^{(j)} = (t_1^{(j)}, t_2^{(j)}, \dots)$). For an element as in (4.1) let us define $\tau_U(t)$ by

$$\tau_U(t) := \frac{\Omega_+(\{\exp(\sum_i t_i^{(j)} z_j^{-i})\} \cdot U)}{\{\exp(\sum_i t_i^{(j)} z_j^{-i})\} \cdot \delta_U}$$

where δ_U is a non-zero element in the fibre of Det_V^* over U .

Let z denote $(z_1, \dots, z_r)$. For $1 \leq u, v \leq r$, let $[z_v] := (z_v, \frac{z_v^2}{2}, \frac{z_v^3}{3}, \dots)$, $t + [z_v] := (t^{(1)}, \dots, t^{(v)} + [z_v], \dots, t^{(r)})$ and $U_{uv} := (1, \dots, z_u, \dots, 1) \cdot (1, \dots, z_v^{-1}, \dots, 1) \cdot U$.

The u -th Baker-Akhiezer function of a point $U \in \mathrm{Gr}^0(V)$ is the V -valued function

$$\psi_{u,U}(z, t) := (\psi_{u,U}^{(1)}(z_1, t), \dots, \psi_{u,U}^{(r)}(z_r, t)) \in V \widehat{\otimes}_k \mathcal{O}(\mathcal{J}(\widehat{C}_V))$$

where

$$\psi_{u,U}^{(v)}(z_v, t) := \exp\left(-\sum_{i \geq 1} \frac{t_i^{(v)}}{z_v^i}\right) \frac{\tau_{U_{uv}}(t + [z_v])}{\tau_U(t)}.$$

Observe that, since V is a finite $k((z))$ -algebra, there is a natural pairing

$$(4.4) \quad \begin{aligned} V \times V &\longrightarrow k \\ (a, b) &\longmapsto \operatorname{Res}_{z=0} \operatorname{Tr}(ab)dz \end{aligned}$$

where $\operatorname{Tr} : V \rightarrow k((z))$ is the trace map. Since this pairing is non-degenerate, we have an involution of $\operatorname{Gr}(V)$ which maps a point U to its orthogonal $U^\perp$. The u -th adjoint Baker-Akhiezer function of U is defined by

$$\psi_{u,U}^*(z, t) := \psi_{u,U^\perp}(z, -t) .$$

Note that one has the following identity

$$\psi_{u,U}^{*(v)}(z_v, t) = \exp\left(\sum_{i \geq 1} \frac{t_i^{(v)}}{z_v^i}\right) \frac{\tau_{U_{vu}}(t - [z_v])}{\tau_U(t)} .$$

THEOREM 4.1. *Let U be a rational point in $\operatorname{Gr}^0(V)$, $1 \leq u \leq r$ and $g \in \operatorname{QP}_r^m$. Then, the following relation holds true*

$$\psi_{u,g(U)}(z, t) = \psi_{u,U}(g(z), (g^*)^{-1}(t)) .$$

PROOF. Since the computation of the Baker-Akhiezer function requires to deal simultaneously with elements in V and with the variables of Baker-Akhiezer function, we should make a distinction between them. Let us introduce the notation $\bar{V} := k((\bar{z}_1)) \times \cdots \times k((\bar{z}_r))$ for a $k((z))$ -algebra isomorphic to V .

Then, Baker-Akhiezer functions are $\bar{V}$ -valued functions on $\mathcal{J}(\widehat{C}_V)$. The v -th component of the u -th Baker-Akhiezer function of $g(U)$ (where $1 \leq u, v \leq r$), which might be thought as the factor of $\psi_{u,g(U)}(\bar{z}, t)$ which depends on the variable $\bar{z}_v$, is

$$\psi_{u,g(U)}^{(v)}(\bar{z}_v, t) = \exp\left(-\sum_i \frac{t_i^{(v)}}{\bar{z}_v^i}\right) \frac{\tau_{g(U)_{uv}}(\{\exp(\sum_i (t_i^{(j)} + \frac{1}{i} \delta_v^j \bar{z}_v^i) z_j^{-i})\}_{1 \leq j \leq r})}{\tau_{g(U)}(\{\exp(\sum_i t_i^{(j)} z_j^{-i})\}_{1 \leq j \leq r})}$$

Let $g = (a_{ij})$ be in QP_r^m . Consider a permutation $\sigma \in \mathcal{S}_r$ and roots of unity $\{\epsilon_j\}$ such that $g(z_j) = \epsilon_{\sigma(j)} z_{\sigma(j)}$. Then, the automorphism g is the composition of the automorphism defined by σ , e.g. $\sigma(z_j) := z_{\sigma(j)}$, followed by the automorphism ϵ defined by $\epsilon(z_j) := \epsilon_j z_j$. Thus, it suffices to show the statement for these two types of automorphisms.

Let us deal with the first case; that is, for the permutation matrix corresponding to σ . Denoting $U_{\sigma^{-1}(u)\sigma^{-1}(v)}$ by U' , one has that $\psi_{u,\sigma(U)}^{(v)}(\bar{z}_v, t)$ equals

$$\begin{aligned}
& \exp\left(-\sum_i \frac{t_i^{(v)}}{\bar{z}_v^i}\right) \frac{\tau_{U'}(\{\exp(\sum_i (t_i^{(j)} + \frac{1}{i} \delta_v^j \bar{z}_v^i) \sigma^{-1}(z_j^{-i}))\}_{1 \leq j \leq r})}{\tau_U(\{\exp(\sum_i t_i^{(j)} \sigma^{-1}(z_j^{-i}))\}_{1 \leq j \leq r})} = \\
& = \exp\left(-\sum_i \frac{t_i^{(v)}}{\bar{z}_v^i}\right) \frac{\tau_{U'}(\{\exp(\sum_i (t_i^{(j)} + \frac{1}{i} \delta_v^j \bar{z}_v^i) z_{\sigma^{-1}(j)}^{-i})\}_{1 \leq j \leq r})}{\tau_U(\{\exp(\sum_i t_i^{(j)} z_{\sigma^{-1}(j)}^{-i})\}_{1 \leq j \leq r})} = \\
& = \exp\left(-\sum_i \frac{t_i^{(v)}}{\bar{z}_v^i}\right) \frac{\tau_{U'}(\{\exp(\sum_i (t_i^{(\sigma(j))} + \frac{1}{i} \delta_v^{\sigma(j)} \bar{z}_v^i) z_j^{-i})\}_{1 \leq j \leq r})}{\tau_U(\{\exp(\sum_i t_i^{(\sigma(j))} z_j^{-i})\}_{1 \leq j \leq r})} = \\
& = \exp\left(-\sum_i \frac{(\sigma^*)^{-1}(t_i^{(\sigma^{-1}(v))})}{\sigma(\bar{z}_{\sigma^{-1}(v)}^i)}\right) \cdot \\
& \quad \cdot \frac{\tau_{U'}(\{\exp(\sum_i ((\sigma^*)^{-1}(t_i^{(j)}) + \frac{1}{i} \delta_{\sigma^{-1}(v)}^j \sigma(\bar{z}_{\sigma^{-1}(v)}^i)) z_j^{-i})\}_{1 \leq j \leq r})}{\tau_U(\{\exp(\sum_i (\sigma^*)^{-1}(t_i^{(j)}) z_j^{-i})\}_{1 \leq j \leq r})} = \\
& = \psi_{\sigma^{-1}(u), U}^{(v)}(\sigma(\bar{z}_{\sigma^{-1}(v)}), (\sigma^*)^{-1}(t))
\end{aligned}$$

which should be thought as the factor of $\psi_{\sigma^{-1}(u), U}(\sigma(\bar{z}), (\sigma^*)^{-1}(t))$ depending on z_v .

It remains to check the formula for the automorphisms ϵ of the type $\epsilon(z_j) := \epsilon_j z_j$ where ϵ_j are m -th roots of unity. However, the proof for this type of automorphisms can be easily deduced from the Proposition 5.10 of [SW] (where the case $m = r = 1$ is carried out) or, alternatively, from the arguments of the proof of the Theorem 3.10 of [MP2].

Recall that the image under the Krichever map of a point $(Y, \bar{t}, t_{\bar{y}}) \in \mathcal{M}^\infty(r)$, where Y is connected of genus g_Y , lies on the connected component of $\text{Gr}(V)$ of index $1 - g_Y$. Therefore, the above result have to be extended to the whole Grassmannian.

From [MP2], we know that the involution $U \mapsto U^\perp$, which maps a subspace to its orthogonal with respect to the pairing (4.4), sends the connected component $\text{Gr}^l(V)$ to $\text{Gr}^{l'}(V)$ where $l' := r(1 - m) - l$. In order to extend the section Ω_+ to $\text{Gr}(V)$, we fix elements $\varepsilon_l \in V^+$ for $l > \frac{1}{2}r(1 - m)$ such that $\dim V^+ / (V^+ \cap \varepsilon_l V^+) - \dim \varepsilon_l V^+ / (V^+ \cap \varepsilon_l V^+) = l$. For $l < \frac{1}{2}r(1 - m)$, set ε_l by the rule $\varepsilon_l := \frac{z}{z} \cdot \varepsilon_{l'}^{-1}$ where $z = (z_1, \dots, z_r)$, $z = (z_1^m, \dots, z_r^m)$ and $l' = r(1 - m) - l$.

Once the elements $\{\varepsilon_l\}$ have been fixed, we define $\Omega_+(U) := \Omega_+(\varepsilon_l^{-1}U)$ for $U \in \text{Gr}^l(V)$. Now, τ -functions, BA-functions and adjoint BA-functions are defined for any point $U \in \text{Gr}^l(V)$ by the same formulae and, in particular, for $U \in \text{Gr}^l(V)$ it holds that

$$\psi_{u, U}(z, t) = \psi_{u, \varepsilon_l^{-1}U}(z, t) \cdot$$

This identity and the Theorem 4.1 allows us to compute $\psi_{u,g(U)}(z, t)$ for $g \in \mathbf{QP}_r^m$ as follows

$$\begin{aligned}
 \psi_{u,g(U)}(z, t) &= \psi_{u,\varepsilon_l^{-1}g(U)}(z, t) = \psi_{u,g(g^{-1}(\varepsilon_l^{-1})U)}(z, t) = \\
 &= \psi_{u,g^{-1}(\varepsilon_l^{-1})U}(g(z), (g^*)^{-1}(t)) = \\
 (4.5) \quad &= \psi_{u,g^{-1}(\varepsilon_l^{-1})\varepsilon_l\varepsilon_l^{-1}U}(g(z), (g^*)^{-1}(t)) = \\
 &= \frac{\varepsilon_l}{g^{-1}(\varepsilon_l)} \cdot \psi_{u,\varepsilon_l^{-1}U}(g(z), (g^*)^{-1}(t)) = \\
 &= \frac{\varepsilon_l}{g^{-1}(\varepsilon_l)} \cdot \psi_{u,U}(g(z), (g^*)^{-1}(t)) .
 \end{aligned}$$

Recall that the image of $\mathrm{Gr}(V)$ into a infinite dimensional projective space by the Plücker map is characterized by bilinear identities ([MP2]). More precisely, for $U, U' \in \mathrm{Gr}^l(V)$ it holds that $U = U'$ if and only if

$$\mathrm{Res}_{z=0} \mathrm{Tr} \frac{z \cdot \psi_{u,U'}(z, t)}{(1, \dots, z_u, \dots, 1)}, \frac{\psi_{v,U}^*(z, t')}{(1, \dots, z_v, \dots, 1)} \frac{dz}{z} = 0$$

for all t, t' and $1 \leq u, v \leq r$.

These arguments and formula (4.5) imply that a point $U \in \mathrm{Gr}^l(V)$ satisfies $g(U) = U$ for $g \in \mathbf{QP}_r^m$ if and only if

$$(4.6) \quad \mathrm{Res}_{z=0} \mathrm{Tr} \frac{\varepsilon_l z \cdot \psi_{u,U}(g(z), (g^*)^{-1}(t))}{g^{-1}(\varepsilon_l)(1, \dots, z_u, \dots, 1)}, \frac{\psi_{v,U}^*(z, t')}{(1, \dots, z_v, \dots, 1)} \frac{dz}{z} = 0$$

for all t, t' and $1 \leq u, v \leq r$.

Then, one has the following

THEOREM 4.2. *Let G be a finite group of order $r \cdot m$ and $\rho : G \hookrightarrow \mathbf{QP}_r^m$ as in (2.4). Let $(Y, \bar{y}, t_{\bar{y}}) \in \mathcal{M}^\infty(r)$ with Y smooth and connected and $U \in \mathrm{Gr}(V)$ its image under the Krichever map.*

Then, $(Y, \bar{y}, t_{\bar{y}}) \in \mathcal{M}_{G,\rho}^\infty(r)$ if and only if U satisfies the bilinear equations (4.6) for all $g \in \mathrm{Im}(\rho)$.

PROOF. It follows from the identity $\underline{\mathcal{M}}_{G,\rho}^\infty(r) = \underline{\mathcal{M}}^\infty(r) \times_{\underline{\mathrm{Gr}}(V)} \underline{\mathrm{Gr}}(V)^{\rho(G)}$ and the equation (4.6).

THEOREM 4.3. *Let U be a rational point of $\mathcal{H}^\infty(m, \overset{r}{\cdot}, m)$.*

Then, U lies in $\mathcal{H}_{\mathrm{Gal}}^\infty(m, \overset{r}{\cdot}, m)$ if and only if there exists $g_1, \dots, g_{rm} \in \mathbf{QP}_r^m$ satisfying (4.6).

PROOF. It is straightforward from Theorem 3.4 and equation (4.6).

Finally, observe that the previous theorems give the equations of $\mathcal{M}_{G,\rho}^\infty(r)$ and $\mathcal{H}_{\mathrm{Gal}}^\infty(m, \overset{r}{\cdot}, m)$ into $\mathrm{Gr}(V)$ since the equations of $\mathcal{M}^\infty(r)$ and $\mathcal{H}^\infty(m, \overset{r}{\cdot}, m)$ have been already computed in [MP2]. Furthermore, recall that these equations can be written as hierarchies of p.d.e. for the τ -function.

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