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Plaza Martín, Francisco Jose

Representations of the Witt Algebra and $Gl(n)$ -Ops.

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Muñoz Porras, J. M. (E-SALA); Plaza Martín, F. J. (E-SALA); Serrano Sotelo, G. (E-SALA)

Convolutional Goppa codes defined on fibrations. (English summary)

Appl. Algebra Engrg. Comm. Comput. **23** (2012), no. 3-4, 165–178.1432-0622

{A review for this item is in process.}

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MR2929727 (Review) 14H60 (14D21 17B67 17B68 22E65 22E67)

Gómez González, E. (E-SALA); Hernández Serrano, D. (E-SALA);

Muñoz Porras, J. M. (E-SALA); Plaza Martín, F. J. (E-SALA)

Geometric approach to Kac-Moody and Virasoro algebras. (English summary)

J. Geom. Phys. **62** (2012), no. 9, 1984–1997.

Summary: “In this paper we show the existence of a group acting infinitesimally transitively on the moduli space of pointed-curves and vector bundles (with formal trivialization data) and whose Lie algebra is an algebra of differential operators. The central extension of this Lie algebra induced by the determinant bundle on the Sato Grassmannian is precisely a semidirect product of a Kac-Moody algebra and the Virasoro algebra. As an application of this geometric approach, we give a local Mumford-type formula in terms of the cocycle associated with this central extension. Finally, using the original Mumford formula we show that this local formula is an infinitesimal version

of a general relation in the Picard group of the moduli of vector bundles on a family of curves (without any formal trivialization).”

Reviewed by [Francesco Bottacin](#)

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[MR2657684 \(2012f:37151\)](#) [37K20](#) (14H40)

Gómez González, Esteban (E-SALA); **Muñoz Porras, José M.** (E-SALA);

Plaza Martín, Francisco J. (E-SALA);

Rodríguez, Rubí E. [Rodríguez, Rubí Elena] (RCH-UCC)

Characterizations of Jacobians of curves with automorphisms. (English summary)

Trans. Amer. Math. Soc. **362** (2010), no. 10, 5373–5394.1088-6850

Summary: “We obtain a characterization of theta functions of Jacobian varieties of curves with automorphisms among theta functions of principally polarized abelian varieties (p.p.a.v.). We first give a characterization in terms of finite dimensional orbits for a suitable action in the Sato Grassmannian. Secondly, the introduction of formal Baker-Akhiezer functions and formal τ -functions attached to a p.p.a.v. (for the multipuncture case) allows us to characterize, in terms of bilinear identities, those Baker-Akhiezer functions that are Baker-Akhiezer functions of Jacobians of curves with automorphisms. Further, in the case of automorphisms with fixed points, we rewrite the previous result as a hierarchy of partial differential equations for the τ -function of a p.p.a.v. Finally, since Baker-Akhiezer and τ functions are written in terms of theta functions, these results give rise to characterizations of p.p.a.v. in terms of their theta functions.”

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groups, Adv. Ser. Math. Phys., 7 (Luminy-Marseille, 1988), pp. 407–448, World Sci. Publishing, Teaneck, NJ, 1989. [MR1026961 \(91c:14038\)](#)

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[MR2629642 \(2011k:37117\)](#) 37K20 (14H10 35Q53 37K10)

[Muñoz Porras, J. M. \(E-SALA\)](#); [Plaza Martín, F. J. \(E-SALA\)](#)

Coverings with prescribed ramification and Virasoro groups. (English summary)

Kyoto J. Math. **50** (2010), no. 1, 51–73.

There is a beautiful and well-known relationship between the conformal field theory of free fermions, the KP hierarchy, and the geometry of moduli spaces of curves and line bundles on them, all mediated by the geometry of the Sato Grassmannian. See [T. Miwa, M. Jimbo and E. Date, *Solitons*, translated from the 1993 Japanese original by Miles Reid, Cambridge Tracts in Math., 135, Cambridge Univ. Press, Cambridge, 2000; [MR1736222 \(2001a:37109\)](#); E. V. Frenkel and D. Ben-Zvi, *Vertex algebras and algebraic curves*, second edition, Math. Surveys Monogr., 88, Amer. Math. Soc., Providence, RI, 2004; [MR2082709 \(2005d:17035\)](#); N. Kawamoto et al., Comm. Math. Phys. **116** (1988), no. 2, 247–308; [MR0939049 \(89i:14020\)](#); M. Mulase, in *Perspectives in mathematical physics*, 151–217, Conf. Proc. Lecture Notes Math. Phys., III, Int. Press, Cambridge, MA, 1994; [MR1314667 \(96c:58088\)](#); G. B. Segal and G. Wilson, Inst. Hautes Études Sci. Publ. Math. No. 61 (1985), 5–65; [MR0783348 \(87b:58039\)](#)] for various perspectives.

The paper under review generalizes aspects of this story from moduli of curves to Hurwitz spaces, i.e. moduli of ramified covers of curves $Y \rightarrow X$ with prescribed ramification data. Prior work in this direction appears in [J. M. Muñoz Porras and F. J. Plaza Martín, Math. Nachr. **281** (2008), no. 7, 989–1012; [MR2431573 \(2009g:14036\)](#)] in the case of one point of ramification, but the present paper handles the case of general ramification profiles.

In particular, the authors describe the “formal Hurwitz scheme” (i.e. the analog of the Hurwitz scheme that equips Y and X with compatible formal coordinates at the ramification points) as an infinite-dimensional subscheme of the Sato Grassmannian. They also compute the tangent space to the Hurwitz scheme at a given point, and use this computation to give a uniformization of the Hurwitz scheme analogous to the usual Virasoro uniformization of the moduli space of curves. Finally, the story is extended to the universal Picard variety over the formal Hurwitz scheme. The paper also gives a characterization of the curves that appear as ramified covers of another curve, with prescribed ramification data, in terms of bilinear identities that must be satisfied by the

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Plaza Martín, Francisco J. (E-SALA)

Arithmetic infinite Grassmannians and the induced central extensions. (English summary)

Collect. Math. **61** (2010), *no. 1*, 107–129.

Infinite-dimensional Grassmann manifolds were originally introduced by M. Sato and Y. Sato [in *Nonlinear partial differential equations in applied science (Tokyo, 1982)*, 259–271, North-Holland Math. Stud., 81, North-Holland, Amsterdam, 1983; [MR0730247 \(86m:58072\)](#)] in the study of solutions of the KP hierarchy.

More recently, an algebro-geometric construction of infinite Grassmannians, parametrizing suitable subspaces of an infinite-dimensional vector space over an arbitrary field k , was given in [A. Álvarez Vázquez, J. M. Muñoz Porras and F. J. Plaza Martín, in *Workshop on Abelian Varieties and Theta Functions (Spanish) (Morelia, 1996)*, 3–40, Aportaciones Mat. Investig., 13, Soc. Mat. Mexicana, México, 1998; [MR1781698 \(2002d:14051\)](#)].

From the point of view of algebraic geometry, the next natural generalization consists in performing the construction of infinite Grassmannians in families, i.e., in replacing the ‘base scheme’ $\text{Spec}(k)$ with an arbitrary scheme S . This is the construction presented in the paper under review. More precisely, given an arbitrary scheme S , the author constructs an infinite Grassmannian parametrizing suitable submodules of a given $\mathcal{O}_S$ -module V : this Grassmannian turns out to be a separated scheme over S .

We remark that the problem of the construction of infinite Grassmannians in a relative setting had been previously investigated by other authors; see, for instance, [I. Quandt, Bayreuth. Math. Schr. No. 52 (1997), 1–74; [MR1453427 \(98h:14023\)](#)].

The second main result obtained in the present paper is the construction of the determinant line bundle on the infinite Grassmannian. This is achieved by using the classical construction of the determinant of a perfect complex of sheaves [F. F. Knudsen and D. B. Mumford, *Math. Scand.* **39** (1976), no. 1, 19–55; [MR0437541 \(55 #10465\)](#)].

Finally, the paper ends with a discussion of some extensions (of a group by a monoid) naturally related to the previous construction of the determinant line bundle on the infinite Grassmannian.

Reviewed by [Francesco Bottacin](#)

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Hernández-Serrano, D. [**Hernández Serrano, D.**] (E-SALA);

Muñoz Porras, J. M. (E-SALA); **Plaza Martín, F. J.** (E-SALA)

Equations of the moduli of Higgs pairs and infinite Grassmannian. (English summary)

Internat. J. Math. **20** (2009), no. 8, 1029–1055.1793-6519

Let X be a smooth, projective, integral curve defined over an algebraically closed field k of characteristic 0, let us choose a point $x \in X$ and fix a formal trivialization $t: \widehat{\mathcal{O}}_{X,x} \xrightarrow{\sim} k[[z]]$. We shall denote by $\mathcal{U}_X^\infty$ the moduli space parametrizing pairs consisting of a rank n vector bundle E over X and a formal trivialization ϕ of E at the point x : $\phi: \widehat{E}_x \xrightarrow{\sim} \widehat{\mathcal{O}}_{X,x}^{\oplus n}$. The moduli space $\mathcal{U}_X^\infty$ can be embedded as a closed subscheme of an infinite Grassmannian by means of an analogue of the classical Krichever map, $\mathcal{U}_X^\infty \hookrightarrow \text{Gr}(V)$, where $V = k((z))^n$ (we refer to [M. Mulase, *Internat. J. Math.* **1** (1990), no. 3, 293–342; [MR1078516 \(92d:58083\)](#)] for the definitions and the construction

of the Krichever embedding).

Given a vector bundle E over X , a Higgs field is a homomorphism of $\mathcal{O}_X$ -modules $\varphi: E \rightarrow E \otimes \omega_X$, where ω_X is the canonical line bundle on X . We shall denote by Higgs_X^∞ the moduli space parametrizing isomorphism classes of triples (E, ϕ, φ) , where $(E, \phi) \in \mathcal{U}_X^\infty$ and φ is a Higgs field (it is also required that the Higgs field φ and the formal trivialization ϕ satisfy a certain compatibility condition). In this context there is a formal analogue of the Hitchin map that associates to a triple (E, ϕ, φ) as above, the coefficients of the characteristic polynomial of the formal completion $\widehat{\varphi}$, considered as elements of the formal power series ring $k[[z]]$. If we denote by $\mathbb{A}^\infty$ the infinite-dimensional affine space over k , then the formal Hitchin map is a morphism of schemes $H_\infty: \text{Higgs}_X^\infty \rightarrow \mathbb{A}$, where $\mathbb{A} = (\mathbb{A}^\infty)^n$ is the formal analogue of the Hitchin base space.

Using these results, the authors prove that the moduli space Higgs_X^∞ can be embedded, by means of an analogue of the Krichever map, as a closed subscheme of the product $\text{Gr}(V) \times \mathbb{A}$; moreover, the image of this embedding can be described in an explicit way. Then, using this characterization, they are able to compute explicit KP-type equations for the moduli space Higgs_X^∞ . Finally, the last section is devoted to the analysis of the case in which the spectral cover is totally ramified at a fixed point of X . In this case, it is shown that the equations obtained in the previous section can be written in terms of the characteristic coefficients of the Higgs field.

Reviewed by [Francesco Bottacin](#)

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Casimiro, A. C. [Casimiro, Ana Cristina] (P-NULST); **Muñoz Porras, J. M.** (E-SALA); **Plaza Martín, F. J.** (E-SALA)

Quotients on the Sato Grassmannian and the moduli of vector bundles. (English summary)
J. Phys. A **41** (2008), no. 19, 194004, 10 pp. 1751–8121

Let us consider the following infinite-dimensional $\mathbb{C}$ -vector spaces, $V = \mathbb{C}((z))^{\oplus r}$ and $V_+ = \mathbb{C}[[z]]^{\oplus r}$, and denote by $\text{Gr}(V, V_+)$ the infinite Grassmannian parametrizing vector subspaces of V that are “commensurable” with V_+ (see [A. Álvarez Vázquez, J. M. Muñoz Porras and F. J. Plaza Martín, in *Workshop on Abelian Varieties and Theta Functions (Spanish) (Morelia, 1996)*, 3–40, Soc. Mat. Mexicana, México, 1998; [MR1781698 \(2002d:14051\)](#)] for the precise definition).

The infinite Grassmannian $\mathrm{Gr}(V, V_+)$ can be related to the usual finite-dimensional Grassmannians as follows: $\mathrm{Gr}(V, V_+)$ is a union of an increasing family of open subsets U_m , for all $m > 0$, where each U_m can be expressed as an inverse limit of open subsets of finite Grassmannians.

Now let us consider the action of the reductive group $\mathrm{SL}(r, \mathbb{C})$ on $\mathrm{Gr}(V, V_+)$. By using the preceding description of the infinite Grassmannian in terms of finite-dimensional ones, it is possible to introduce a natural notion of stability for points of $\mathrm{Gr}(V, V_+)$ with respect to the action of $\mathrm{SL}(r, \mathbb{C})$. This was done by the same authors in [J. Geom. Phys. **58** (2008), no. 3, 402–421; [MR2394045 \(2009b:14094\)](#)]. In this context one is then naturally led to wonder about the existence of quotients.

In the paper under review, the authors prove that the set of stable points of $\mathrm{Gr}(V, V_+)$ admits a geometric quotient for the action of $\mathrm{SL}(r, \mathbb{C})$. This result is obtained by relating the existence of such a quotient to the existence of a quotient of the set of stable points of finite-dimensional Grassmannians for the action of the same reductive group $\mathrm{SL}(r, \mathbb{C})$. It turns out, in fact, that the quotient of the set of stable points of $\mathrm{Gr}(V, V_+)$ for the action of $\mathrm{SL}(r, \mathbb{C})$ can be constructed as an infinite union of an increasing family of open subsets, each one constructed as an inverse limit of quotients of (open subsets of) finite Grassmannians.

The last section of the paper is then devoted to the analysis of the relation between the infinite Grassmannian $\mathrm{Gr}(V, V_+)$ and the moduli spaces of vector bundles over a smooth, irreducible, projective curve C over $\mathbb{C}$, equipped with a trivialization over a point of C .

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Muñoz Porras, José M. (E-SALA); Plaza Martín, Francisco J. (E-SALA)

Equations of Hurwitz schemes in the infinite Grassmannian. (English summary)

Math. Nachr. **281** (2008), no. 7, 989–1012.

The problem of extending the classical Krichever map to the case of finite coverings of pointed Riemann surfaces with trivializations at the punctures has been studied by several authors. In the paper under review, the authors use the extended Krichever functor to embed Hurwitz schemes of coverings of Riemann surfaces as closed subschemes of Sato's infinite Grassmannian. More precisely, let V be a finite separable $\mathbb{C}((z))$ -algebra of dimension n and $V^+ \subset V$ a $\mathbb{C}[[z]]$ -subalgebra of rank n over $\mathbb{C}[[z]]$, and let us denote by $\mathrm{Gr}(V, V^+)$ the corresponding infinite Grassmannian of Sato. Now, let us consider positive integers $e_1, \dots, e_r$, with $e_1 + \dots + e_r = n$, and let us denote by $H(e_1, \dots, e_r)$ the Hurwitz scheme parametrizing geometric data $(Y, X, \pi, x, \bar{y}, t_x, t_{\bar{y}})$, where X and Y are Riemann surfaces, $\pi: Y \rightarrow X$ is a finite covering of degree n , $x \in X$, $\bar{y} = \pi^{-1}(x) = e_1 y_1 + \dots + e_r y_r$ and $t_x, t_{\bar{y}}$ are formal parameters at x and $\bar{y}$, respectively.

After recalling basic results about the infinite Grassmannian $\mathrm{Gr}(V, V^+)$, the authors prove that by letting $V = \mathbb{C}((z^{1/e_1})) \times \cdots \times \mathbb{C}((z^{1/e_r}))$, one can construct an immersion (the Krichever map) $K: H(e_1, \dots, e_r) \rightarrow \mathrm{Gr}(V, V^+)$. By using this map, the Hurwitz space of coverings $H(e_1, \dots, e_r)$ can be considered as a closed subscheme of $\mathrm{Gr}(V, V^+)$. Then, by extending to the current setting a series of results about infinite Grassmannians developed in [A. Álvarez Vázquez, J. M. Muñoz Porras and F. J. Plaza Martín, in *Workshop on Abelian Varieties and Theta Functions (Spanish) (Morelia, 1996)*, 3–40, Soc. Mat. Mexicana, México, 1998; [MR1781698 \(2002d:14051\)](#)] and [J. M. Muñoz Porras and F. J. Plaza Martín, *J. Differential Geom.* **51** (1999), no. 3, 431–469; [MR1726736 \(2001a:14032\)](#)], it is possible to find an explicit set of equations describing the closed subscheme $H(e_1, \dots, e_r)$ of $\mathrm{Gr}(V, V^+)$. By considering the τ -functions associated to points of $\mathrm{Gr}(V, V^+)$, these equations can be translated into a hierarchy of differential equations. As a final application, the authors provide a characterization of the existence of certain linear series on a smooth curve in terms of soliton equations.

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Casimiro, A. C. [Casimiro, Ana Cristina] (P-NULST); **Muñoz Porras, J. M.** (E-SALA); **Plaza Martín, F. J.** (E-SALA)

Stability on the Sato Grassmannian. Applications to the moduli of vector bundles. (English summary)

J. Geom. Phys. **58** (2008), no. 3, 402–421.

This paper is devoted to the study of the notion of stability of vector bundles over a smooth algebraic curve in terms of Sato's infinite Grassmannian [cf. also D. V. Osipov, Mat. Sb. **194**

(2003), no. 11, 81–94; [MR2040666 \(2004m:14068\)](#)]. The link between vector bundles over an algebraic curve C and points of the infinite Grassmannian is provided by the so-called Krichever map, which assigns a point of the Grassmannian to a pair consisting of a vector bundle E over C and a trivialization of E in a formal neighborhood of a point $P \in C$ [see M. Mulase, *Internat. J. Math.* **1** (1990), no. 3, 293–342; [MR1078516 \(92d:58083\)](#)].

In order to fix some notation, let us denote by k an algebraically closed field of characteristic 0 and by V the infinite-dimensional vector space $k((z))^{\oplus r}$. The infinite-dimensional Sato Grassmannian associated to V will be denoted by G (the precise definition of G is recalled in Section 2 of this paper). The group $\mathrm{SL}(r, k[[z]])$ acts naturally on G . The main part of the paper under review is then devoted to the study of this action, with the aim of providing a natural definition of (semi)stable points of G , similar to the one given by the usual GIT theory of Mumford. The main difficulties are due to the fact that the group $\mathrm{SL}(r, k[[z]])$ is neither of finite type nor reductive; moreover, Sato’s Grassmannian is not of finite type.

The authors begin their investigation with the study of the action of the subgroup $\mathrm{SL}(r, k)$ on G . This is done by observing that G can be covered by an ascending chain of open subschemes, where each of these subschemes is an inverse limit of open sets of finite Grassmannians. By studying the action of $\mathrm{SL}(r, k)$ on each of these finite Grassmannians, they arrive at a natural definition of stability on G . Finally, by using these results, they propose a definition of stability for the action of the group $\mathrm{SL}(r, k[[z]])$ on G , and they give a corresponding natural criterion expressed in terms of the “slope” (that recalls the usual criterion of stability for vector bundles). The next step is the study of Harder-Narasimhan and Jordan-Hölder filtrations of points of G . This is carried out in Section 3.

The last section of the paper is devoted to the comparison between the notions of stability, Harder-Narasimhan and Jordan-Hölder filtrations for vector bundles on the curve C and the corresponding notions for the points of the infinite Grassmannian G .

Reviewed by [Francesco Bottacin](#)

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MR2218001 (2007b:14060) [14H37](#) ([14H10](#) [14H30](#) [58B25](#))

Gómez González, E. (E-SALA); **Plaza Martín, F. J.** (E-SALA)

Curves with a group action and Galois covers via infinite Grassmannians. (English summary)

The geometry of Riemann surfaces and abelian varieties, 105–117, *Contemp. Math.*, 397, Amer. Math. Soc., Providence, RI, 2006.

In the paper under review, the theory of Sato's infinite Grassmannian is used to investigate the moduli functor of smooth curves with the action of a finite group G . Specifically, the authors consider the moduli functor $\mathcal{M}_G^\infty(r)$ of curves with an action of G , a distinguished G -orbit of cardinality r and a formal trivialization at the points of the orbit, together with a group monomor-

phism from G to the group of $r \times r$ matrices of quasipermutations with coefficients in the group of m th roots of unity. First, the functor $\mathcal{M}_G^\infty(r)$ is proved to be representable by a subscheme of the infinite Grassmannian. Subsequently, a geometric characterization of this subscheme is given, as well as bilinear equations defining it inside the infinite Grassmannian. The techniques are similar to those used by the authors and J. M. Muñoz Porras in [Math. Ann. **327** (2003), no. 4, 609–639; [MR2023311 \(2004m:14053\)](#)] for the special case of curves with an automorphism of prime order. Furthermore, analogous results are produced for the moduli space of Galois covers.

This is an interesting paper, which would have benefited from a more accurate editing of the material.

{For the entire collection see [MR2218504 \(2006m:14001\)](#)}

Reviewed by [Orsola Tommasi](#)

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MR2194785 (2006k:14088) 14M15 (58B99)

Plaza Martín, Francisco J. (E-SALA)

Vector fields on the Sato Grassmannian. (English summary)

Collect. Math. **56** (2005), no. 3, 265–274.

An explicit basis of the space of global vector fields on the Sato Grassmannian is computed and the vanishing of the first cohomology group of the sheaf of derivations is shown.

Reviewed by [Samwel Dalalyan](#)

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MR2023311 (2004m:14053) 14H40 (14H37 14H70 37K20 58B25)

Gómez González, E. (E-SALA); **Muñoz Porras, J. M.** (E-SALA);

Plaza Martín, F. J. (E-SALA)

Prym varieties, curves with automorphisms and the Sato Grassmannian. (English summary)

Math. Ann. **327** (2003), no. 4, 609–639.

The authors declare that the results of their paper are intended to be a first step to solving a Schottky-type problem for curves with automorphisms. The basic concepts used in the paper are the formal spectral covers and their formal Jacobians. First, some results of T. Shiota [in *Infinite-dimensional Lie algebras and groups (Luminy-Marseille, 1988)*, 407–448, World Sci. Publishing, Teaneck, NJ, 1989; [MR1026961 \(91c:14038\)](#)] and F. J. Plaza Martín [Internat. J. Math. **9** (1998), no. 1, 75–93; [MR1612314 \(99a:14037\)](#)] on Prym varieties of curves with an involution are generalized to the general case of an arbitrary automorphism of prime order. Secondly, the equations defining the moduli space of curves with an automorphism of prime order as a subscheme of the Sato Grassmannian are given.

Reviewed by [Samwel Dalalyan](#)

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MR2007605 (2004m:14015) 14D20 (14H60 14H70 14K25)

Gómez González, E. (E-SALA); Plaza Martín, F. J. (E-SALA)

Addition formulae for non-abelian theta functions and applications. (English summary)

J. Geom. Phys. **48** (2003), no. 2-3, 480–502.

The authors consider the generalized theta functions on the moduli space $U_C(r, d)$ of semistable vector bundles over a curve C of genus $g \geq 2$. They prove a generalization of the addition formula for theta functions on abelian varieties. More precisely, let Θ_F be a generalized theta function on $U_C(r, d)$. Then they give in Theorem 3.8 a formula for

$$\frac{\Theta_F(M)}{\Theta_F(M \otimes \mathcal{O}_C(\sum (x_i - y_i)))},$$

where the x_i and y_i are points on the curve C and M is a point in the moduli space $U_C(r, d)$. This theorem is proven by isomorphisms of (pull backs) of theta bundles.

The result is applied to the context of the multicomponent KP hierarchy and to the study the behaviour of generalized theta functions under direct and inverse image by a cyclic covering.

Reviewed by *Georg Hein*

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MR1815719 (2002b:14041) [14H81](#) ([14H70](#) [14L05](#) [81T30](#))

Muñoz Porras, J. M. (E-SALA); **Plaza Martín, F. J.** (E-SALA)

Automorphism group of $k((t))$: applications to the bosonic string. (English summary)

Comm. Math. Phys. **216** (2001), no. 3, 609–634.

D. Mumford [*Enseignement Math.* (2) **23** (1977), no. 1-2, 39–110; [MR0450272 \(56 #8568\)](#)] proved the remarkable formula

$$\lambda_n \simeq \lambda_1^{\otimes (6n^2 - 6n + 1)}$$

for the determinant bundles λ_k , $k \in \mathbb{Z}$, on the moduli space $\mathcal{M}_g$ of curves of genus g . Here λ_k denotes the determinant line bundle of the vector bundle on $\mathcal{M}_g$ whose fiber over a curve C is the space of differentials of degree k on C .

The main result of the paper under review is a proof of an analogous formula for “determinant bundles” Λ_k on a “universal moduli space” related to the infinite-dimensional (Sato) Grassmannian

[E. Arbarello et al., *Comm. Math. Phys.* **117** (1988), no. 1, 1–36; [MR0946992 \(89i:14019\)](#)]. The authors also discuss applications of this “local Mumford formula” to string theory.

Reviewed by *Igor Yu. Potemine*

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MR1794521 (2001j:58009) 58B25

Plaza-Martín, Francisco J. (E-SALA)

The Grassmannian of $k((z))$: Picard group, equations and automorphisms. (English summary)

J. Math. Kyoto Univ. **40** (2000), no. 3, 567–580.

Let G be a connected simply connected simple affine algebraic group over $\mathbf{C}$. S. Kumar, M. S. Narasimhan and A. Ramanathan [*Math. Ann.* **300** (1994), no. 1, 41–75; [MR1289830 \(96e:14011\)](#)] proved that the Picard group of the loop Grassmannian L_G/L_G^+ is isomorphic to $\mathbf{Z}$.

In the paper under review the author extends this result to the case of Sato Grassmannians (related to the group GL_∞). The Plücker equations and automorphisms of Sato Grassmannians are also discussed.

Reviewed by *Igor Yu. Potemine*

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MR1783692 (2001g:14058) 14H70 (14H42 37K20)

Plaza Martín, F. J. (E-SALA)

Algebraic solutions of the multicomponent KP hierarchy. (English summary)

J. Geom. Phys. **36** (2000), no. 1-2, 1–21.

This paper is an important generalization of the author's theory, jointly with J. M. Muñoz Porras, for a scheme-theoretic version of the Krichever map [J. M. Muñoz Porras and F. J. Plaza Martín, *J. Differential Geom.* **51** (1999), no. 3, 431–469; [MR1726736 \(2001a:14032\)](#)]. The Krichever map associates to an algebraic curve C together with a rank- r vector bundle on it (and some additional choices) a point of an infinite-dimensional Grassmannian [cf. M. Sato, in *Theta functions—Bowdoin 1987, Part 1* (Brunswick, ME, 1987), 51–66, *Proc. Sympos. Pure Math.*, 49, Part 1, Amer. Math. Soc., Providence, RI, 1989; [MR1013125 \(91d:58107\)](#)]. The previous work treated the case $r = 1$; here the author adapts the construction to a vector Grassmannian and the attendant “multi-component” KP flows [cf., e.g., M. R. Adams and M. Bergvelt, *Comm. Math. Phys.* **154** (1993), no. 2, 265–305; [MR1224080 \(94k:58055\)](#); V. G. Kac and J. W. van de Leur, in *Important developments in soliton theory*, 302–343, Springer, Berlin, 1993; [MR1280480 \(96b:58051\)](#)]. The key point in relating the formal Krichever map and the KP hierarchy consists in showing that the determinant bundle on the Grassmannian pulls back to the Poincaré bundle on the Jacobian, and the main point of the adaptation to higher rank is to replace the theta divisor on the Jacobian of the curve with the analogous “nonabelian” theta divisor [cf. J.-M. Drézet and M. S. Narasimhan, *Invent. Math.* **97** (1989), no. 1, 53–94; [MR0999313 \(90d:14008\)](#)], again defined by the determinant bundle.

Although in higher rank no explicit formulas for KP solutions exist, the author shows that the solutions can be constructed in terms of theta functions and indicates the possibility that this will give a characterization of nonabelian theta functions. Addition formulas are also derived.

Reviewed by [Emma Previato](#)

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MR1726736 (2001a:14032) [14H70](#) ([14F05](#) [14H42](#) [14L15](#) [22E65](#) [37K20](#))

Muñoz Porras, J. M. (E-SALA); **Plaza Martín, F. J.** (E-SALA)

Equations of the moduli of pointed curves in the infinite Grassmannian. (English summary)

J. Differential Geom. **51** (1999), no. 3, 431–469.

The Krichever map originated with the theory of algebro-geometric solutions to the Kadomtsev–Petviashvili (KP) hierarchy. The input of the map consists of the following data: a curve C defined over $\mathbf{C}$, a smooth point $p \in C$, a local parameter z at p , a (general enough) torsion-free sheaf $\mathcal{L}$ on C and a local trivialization of $\mathcal{L}$; the output is a point U of an infinite Grassmannian, which roughly speaking is the space of sections of $\mathcal{L}$ over $C \setminus \{p\}$.

The authors develop a powerful algebraic formalism which generalizes the theory to arbitrary

fields. On one hand, they reproduce Sato's formal Grassmannian and determinant bundle; on the other, they encode functions on the curve by the use of formal groups. The main idea is an algebraization of the group Γ that acts on the Grassmannian (and is responsible for the KP flows) (cf. [G. Segal and G. Wilson, *Inst. Hautes Études Sci. Publ. Math.* No. 61 (1985), 5–65; [MR0783348 \(87b:58039\)](#)], where Γ is the set of continuous maps $S^1 \rightarrow \mathbf{C}^*$). Here $\Gamma = \Gamma_- \times \mathbf{G}_m \times \Gamma_+$, where Γ_- has associated ring $k\{\{x_1, x_2, \dots\}\}$. In characteristic 0, the assignment $(x_i \mapsto \text{coeff of } z^{-i} \text{ in the series } \exp(\sum_{j>0} y_j z^{-j}))$ gives an isomorphism $\exp: \mathbf{A}_\infty \rightarrow \Gamma_-$, $\exp^*: k\{\{x_1, x_2, \dots\}\} \rightarrow k\{\{y_1, y_2, \dots\}\}$ and the Abel map of a formal curve is $\varphi: C = \text{Spf}(k[[t]]) \rightarrow \Gamma_-$, $\varphi(t) = 1 + \sum_{i>0} t^i/z^i$, $x_i \mapsto t^i$.

The authors then give a remarkable interpretation for the dual of the determinant bundle (which, suitably restricted, amounts to the Poincaré bundle over the dual of the Jacobian), as well as for the sheaf of Baker-Akhiezer functions. In this context they reduce Fay's trisecant identity to linear algebra, and establish the bilinear identity for the tau function, i.e. the KP hierarchy.

Finally, they find (differential) equations that characterize the image of the Krichever map in the infinite Grassmannian. Written in terms of the Baker function $\psi_U(z, t)$, where as customary in the theory $\psi_U^*(z, t)$ denotes the dual Baker function, they are of two types: (1) the KP hierarchy, $\text{Res}_{z=0} \psi_U(z, t) \cdot \psi_U^*(z, t') dz/z^2 = 0$; and (2) $\text{Res}_{z=0} \psi_U(z, t) \cdot \psi_U(z, t') \cdot \psi_U^*(z, t'') dz/z^{2+g} = 0$. More precisely, a formal series $\tau(t) \in k\{\{t_1, t_2, \dots\}\}$ is the tau function associated with a rational point of the image of the Krichever map for curves of genus g in the component of index $g-1$ of the Grassmannian if and only if the associated Baker function satisfies (1), (2), and (3) $\text{Res}_{z=0} \psi_U^*(z, t) dz/z^{2-g} = 0$. These three sets of equations can be equivalently rewritten as differential equations for $\tau_U(t)$, as is customary.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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Álvarez Vázquez, A. (E-SALA-PM); **Muñoz Porras, J. M.** (E-SALA-PM);
Plaza Martín, F. J. (E-SALA-PM)

The algebraic formalism of soliton equations over arbitrary base fields.

Workshop on Abelian Varieties and Theta Functions (Spanish) (Morelia, 1996), 3–40,
Aportaciones Mat. Investig., 13, Soc. Mat. Mexicana, México, 1998.

The aim of this paper is to offer an algebraic construction of infinite-dimensional Grassmannians and determinant bundles. The authors define a functor of points, $\underline{\text{Gr}}(V, \mathcal{T})$, of the Grassmannian of a k -vector space V (with topology $\mathcal{T}$) in such a way that the points $\underline{\text{Gr}}(V, \mathcal{T})(\text{Spec}(k))$ are precisely the Grassmannian defined by Segal-Wilson or Sato-Sato; then they use general theory to show it is representable by a separated k -scheme $\text{Gr}(V, \mathcal{T})$. They use the theory of determinants of Knudsen and Mumford to construct the determinant bundle over $\text{Gr}(V, \mathcal{T})$. The construction of τ -functions and Baker functions is based on an algebraic version of Segal-Wilson theory. A fundamental ingredient, the group Γ of continuous maps $S^1 \rightarrow \mathbb{C}^*$ (elements of it usually represented as series of type $f = \sum_{-\infty}^{+\infty} \lambda_i z^i$, with $\lambda_i \in \mathbb{C}$), is replaced, in their formalism, by a functor whose values in a k -algebra A are the expansions $f = \sum_{i \geq -N} \lambda_i z^i \in A((z))$, such that $\lambda_{-1}, \dots, \lambda_{-N}$ are nilpotent elements of A .

{For the entire collection see [MR1781697 \(2002c:14002\)](#)}

Reviewed by [Luis A. Piovan](#)

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Plaza Martín, Francisco J. (E-SALA)

Prym varieties and the infinite Grassmannian. (English summary)

Internat. J. Math. **9** (1998), no. 1, 75–93.

The author, jointly with J. M. Muñoz Porras [“Equations of the moduli of pointed curves in the infinite Grassmannian”, Preprint, <http://xxx.lanl.gov/abs/alg-geom/9711022>], has developed a scheme-theoretic version of the Krichever map, which associates to an algebraic curve C together with other data a point of an infinite-dimensional Grassmannian [cf. M. Sato, in *Theta functions—Bowdoin 1987, Part 1* (Brunswick, ME, 1987), 51–66, Proc. Sympos. Pure Math., 49, Part 1, Amer. Math. Soc., Providence, RI, 1989; [MR1013125 \(91d:58107\)](#)]. They defined various Krichever

functors, with the result of embedding in the Grassmannian either the moduli space of pointed curves, or the object $\tilde{\text{Pic}}(C, p)$ consisting of the pairs $L \in \text{Pic}(C)$ and a formal trivialization of L near $p \in C$. In this paper, the embedding is generalized to Prym varieties. Although the KP flows on the Jacobian have already been generalized to BKP flows on Pryms [cf. T. Shiota, in *Infinite-dimensional Lie algebras and groups (Luminy-Marseille, 1988)*, 407–448, World Sci. Publishing, Teaneck, NJ, 1989; [MR1026961 \(91c:14038\)](#); Y. Li and M. Mulase, *Comm. Anal. Geom.* **5** (1997), no. 2, 279–332; [MR1483982 \(98j:14043\)](#)], this construction is different. An involution on the Grassmannian is used to construct a skew-symmetric form and the Pryms are characterized, inside the sub-Grassmannian of maximal isotropic subspaces, as the finite-dimensional orbits of a suitable subgroup of the KP flows which preserves this sub-Grassmannian. These are defined to be the BKP flows.

This beautiful construction has the advantage of holding over fields of arbitrary characteristic, of giving a characterization of Jacobians and Pryms (defined by involutions with at least one smooth fixed point), and, in characteristic zero, of expressing the characterization as a set of differential equations for a Pfaffian which plays the role of the tau function.

Reviewed by [Emma Previato](#)

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